Vectors $\vec{KL} \equiv v$ and $\vec{QM} \equiv w$. And let $\vec{KQ} = \vec{l}$. Linear spaces (straight lines) generated by 'em – $p\vec{v}$ and $\vec{l} + q\vec{w}$, with $p, q \in \mathbb{R}$.

The square of the distance (generated by $g_{ij} = \delta_{ij}$; or whatever: invariants are invariants):

$$\rho^2(p,q) = (pv_x - l_x - qw_x)^2 + (pv_y - l_y - qw_y)^2 + (pv_z - l_z - qw_z)^2 =$$

$$= (p\vec{v} - \vec{l} - q\vec{w}, p\vec{v} - \vec{l} - q\vec{w}) = (p\vec{v} - q\vec{w}, p\vec{v} - q\vec{w}) + (\vec{l}, \vec{l}) - 2(p\vec{v} - q\vec{w}, \vec{l}).$$

Geometry (in-plane – $\mathbb{R}^2$ and in $\mathbb{R}^3$) is invariant under translations (at least, the geometry you're talking about), so we may assume that one of the vectors starts at the origin.

Extremal (minimal or maximal) distance – zero of first derivative(s):

$$\frac{d\rho^2}{dp} = 2v_x(pv_x - l_x - qw_x) + 2v_y(pv_y - l_y - qw_y) + 2v_z(pv_z - l_z - qw_z) = 0,$$

$$\frac{d\rho^2}{dq} = 2w_x(pv_x - l_x - qw_x) + 2w_y(pv_y - l_y - qw_y) + 2w_z(pv_z - l_z - qw_z) = 0.$$

δ_{ij} you say? g_{ij}? Drop that: who cares?

$$p(v,v) - q(v,w) = (v,l);$$

$$p(v,w) - q(w,w) = (w,l).$$

We see the (almost Gram-Schmidt) matrix

$$\hat{A} = \begin{bmatrix} (v,v) & -(v,w) \\ (w,v) & -(w,w) \end{bmatrix}; \hat{A}^{-1} = \frac{1}{(v,w)^2 - v^2w^2} \begin{bmatrix} -(w,w) & (v,w) \\ -(v,w) & (v,v) \end{bmatrix}$$

By the way, we have the square of parallelogram area, $S^2(\vec{v}, \vec{w}) = v^2w^2 - (v,w)^2 \equiv |v * w|^2$ (and $S^2 > 0$ due to Kauchy-Buniakovsky inequality). Thus,

$$\begin{bmatrix} p \\ q \end{bmatrix} = \frac{1}{S^2(\vec{v}, \vec{w})} \begin{bmatrix} (w,w) & -(v,w) \\ (v,w) & -(v,v) \end{bmatrix} \begin{bmatrix} (v,l) \\ (w,l) \end{bmatrix} =$$

$$= \frac{1}{S^2(v,w)} \begin{bmatrix} (w,w)(v,l) - (v,w)(w,l) \\ (w,v)(v,l) - (v,v)(w,l) \end{bmatrix}$$

Dig the skew-symmetry!

$$\vec{\gamma} = p\vec{v} - q\vec{w} = \frac{1}{S^2(v,w)} \cdot$$

$$\cdot \{[(w,w)(v,l) - (v,w)(w,l)]\, \vec{v} + [(v,v)(w,l) - (w,v)(v,l)]\, \vec{w}\}.$$

And (*leaping through time*)

$$\rho_{min} = \frac{< l, v, w >}{|v * w|} = \frac{V(\vec{l}, \vec{v}, \vec{w})}{|v * w|}.$$

Task: can you deduce $V(\vec{l}, \vec{v}, \vec{w})$ from all possible scalar products of $\vec{v}$, $\vec{w}$ and $\vec{l}$?

Turn the page!

The leap through time (reminder: $S^2 = v^2w^2 - (v,w)^2$),

$$(\vec{\gamma},\vec{l}) = \frac{1}{S^2}\left[w^2(v,l)^2 - (v,w)(w,l)(v,l)v^2(w,l)^2 - (w,v)(v,l)(w,l)\right] =$$
$$= \frac{1}{S^2}\left[w^2(v,l)^2 - 2(v,w)(w,l)(l,v) + v^2(w,l)^2\right].$$

$$\begin{aligned}
(\vec{\gamma},\vec{\gamma}) = \frac{1}{S^4}\Big\{v^2\left[w^4(v,l)^2 + (v,w)^2(w,l)^2 - 2w^2(v,l)(l,w)(w,v)\right] +\\
+ w^2\left[v^4(w,l)^2 + (w,v)^2(v,l)^2 - 2v^2(w,l)(l,v)(v,w)\right] +\\
+2(v,w)\left[w^2v^2(w,l)(l,v) + (v,w)^2(v,l)(w,l) - w^2(w,v)^2(v,l) - v^2(v,w)^2(w,l)\right]\Big\} =\\
= \frac{1}{S^4}\Big\{v^2w^4(v,l)^2 + w^2v^4(w,l)^2 - v^2(v,w)^2(w,l)^2 - w^2(w,v)^2(v,l)^2 -\\
-2v^2w^2(v,w)(w,l)(l,v) + 2(v,w)^2(w,v)(v,l)(l,w)\Big\} =\\
\frac{1}{S^4}\Big\{v^2w^2\left[w^2(v,l)^2 + v^2(w,l)^2 - 2(v,w)(w,l)(l,v)\right] -\\
-(v,w)^2\left[v^2(w,l)^2 + w^2(v,l)^2 - 2(v,w)(w,l)(l,v)\right]\Big\} =\\
= \frac{w^2(v,l)^2 + v^2(w,l)^2 - 2(v,w)(w,l)(l,w)}{S^2}.
\end{aligned}$$

Algebra is *da shit*, yeah baby!

Recall that

$$\rho^2 = (\vec{\gamma},\vec{\gamma}) + (\vec{l},\vec{l}) - 2(\vec{\gamma},\vec{l}),$$

so

$$\begin{aligned}
S^2\rho^2 = l^2S^2 - \left[w^2(v,l)^2 + v^2(w,l)^2 - 2(v,w)(w,l)(l,v)\right]\\
= \left[l^2v^2w^2 - l^2(v,w)^2 - w^2(v,l)^2 - v^2(w,l)^2 + 2(v,w)(w,l)(l,v)\right] =\\
= V^2(\vec{l},\vec{v},\vec{w}).
\end{aligned}$$

Why such an identity for V^2? Since[1]

$$V^2 = \det\hat{V^2} = \det|v,w,l|\det|v,w,l|^t = \det\begin{bmatrix}(v,v) & (v,w) & (v,l)\\(w,v) & (w,w) & (w,l)\\(l,v) & (l,w) & (l,l)\end{bmatrix} =$$

$$= v^2w^2l^2 + (v,w)(w,l)(l,v) + (v,l)(w,v)(l,w) - v^2(w,l)(l,w) - (v,w)(w,v)l^2 - (v,l)w^2(l,v) =$$
$$= v^2w^2l^2 - v^2(w,l)^2 - w^2(l,v)^2 - l^2(v,w) + 2(v,l)(l,w)(w,v)$$

with $\hat{V^2}$ being the real Gram-Schmidt matrix.

That's about it.

[1] ^{t} is matrix transposition: $M^t_{ij} = M_{ji}$.