VOLUME 3

Feynman

LECTURES ON PHYSICS

EXERCISES / 1965

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INTRODUCTION

The present set of exercises is designed to accompany Vol. III of "Feynman Physics." Like the set which goes with Vol. II, it includes homework and exam problems used at Caltech during the years 1963-64. Again I have tried to arrange the problems roughly in order of difficulty within each chapter.

Even more than the preceding set, this set does not represent a final effort; it must grow as the course evolves. As a matter of fact, these problems have been typed before Vol. III was published in its final form. I hope that any discrepancies in notation which will, therefore, certainly exist will be taken as a further indication of the preliminary nature of these problems.

Most of the problems were written up by M. Sands, R. P. Feynman, J. Pine, and myself. The ideas for about three-fourths were suggested by R. P. Feynman. A preliminary editing was done by C. Wilts and myself in the summer of 1963. The final reading and correction of this collection was done by I. Tammaru.

Again it is my pleasure to thank Mrs. F. L. Warren for typing these problems in all the various stages of preparation.

G. Neugebauer

CHAPTER 3

3-1. An imaginary electron interference experiment is described in this chapter and is shown in Figure 3-1. From the interference pattern P_{12} drawn in the figure, one can estimate the wavelength λ to be associated with the amplitude functions φ_1 and φ_2. Call the distance between the centers of the slits $\underline{a}$, and get any other dimensions needed by measuring on the figure.

a) What do you get for λ?

b) Taking the curves given for P_1 and P_2 in the figure, compute what you would expect for the magnitude of P_{12} at the center of the pattern, at the first maximum away from the center, and at the first two minima in the interference pattern. Compare with the curve for P_{12} given in the figure.

3-2. Consider the two-slit electron interference experiment of Problem 1, and assume that the distances from the gun to the slits and from the slits to the wall are very large in comparison with the distance between the slits. Assume also that the width of the slits is very small compared with their separation. Answer the following questions as quantitatively as you can.

a) What happens to the interference pattern of P_{12} if the gun is moved upward the distance D?

b) What happens to the pattern if the distance between the slits is doubled?

c) What happens to the pattern if slit 1 is made twice as wide as slit 2?

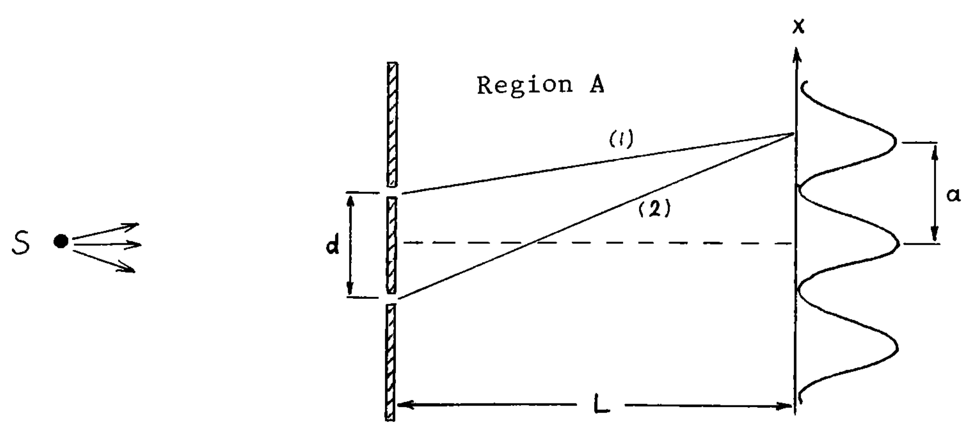
3-3. Vertically polarized monochromatic light is incident on a polaroid whose axis for transmitting light is tilted at an angle θ to the vertical. Classically, what is the ratio of the transmitted intensity to the incident intensity? What does the polaroid do for the case of a single incident photon?

3-4. A beam of 20,000 eV electrons passes through a thin polycrystalline gold foil and then strikes a photographic plate. The plate shows blackening in the form of rings concentric with the axis of the electron beam. Why? Predict the ring diameters, for a distance of 10 cm between gold foil and photographic plate.

3-5. If one considers the standard double slit diffraction experiment shown in Fig. 3-1, it is possible to show that the entire pattern on the screen can be predicted from a knowledge of the amplitudes for the electrons to be at the slits. If a_1 and a_2 are the two complex numbers giving the amplitudes for getting electrons at slits 1 and 2, what is the formula for the relative intensity distribution on the screen as a function of x, the distance from the center point? Make the approximation that x is small; express your answer in terms of the separations between the two slits and between the slits and the viewing screen.

If the pattern depends only on the amplitudes at slits 1 and 2, how does the electron "know" what wavelengths to use behind the slit in determining the pattern?

3-6.



In the diffraction experiment shown in the figure, the source emits particles of momentum p_o, mass M, and velocity $\underline{v}$.

a) What is the spacing "a" between the central maximum and the neighboring maxima? Assume $L \gg d$, $L \gg a$.

b) If some influence alters the phase for the upper path (1) by $\delta\phi_1$ and for the lower path (2) by $\delta\phi_2$, show that the central maximum is displaced by a distance S given by:

$$S = + (\delta\phi_1 - \delta\phi_2) \frac{L}{d} \frac{\hbar}{p_o}$$

Thus, if $(\delta\phi_1 \quad \delta\phi_2)$ is the same for all paths giving rise to the interference pattern, then the whole pattern is displaced and we may say that the particles have been deflected upward a distance S.

c) Suppose that in Region A the particle has a small potential energy which is a function only of its vertical position. Then the momentum of a particle at height $\underline{x}$ above the center line, $p(x)$, will differ slightly from $p(0)$, its value on the center line. Show that $p(x) = p(0) + \frac{M}{p(0)} (V(0) \quad V(x))$, or, for the case where $V(x)$ varies slowly $p(x) = p(0) + \frac{Fx}{v}$, where F is the negative of the gradient of the potential, $-\partial V/dx$.

d) Under the circumstances in (c), the momenta on paths (1) and (2) will differ, and also the wavelengths. Show that the phase difference between the upper and lower paths is:

$$(\delta\phi_1 - \delta\phi_2) = \frac{d}{2v} \frac{F}{\hbar} L$$

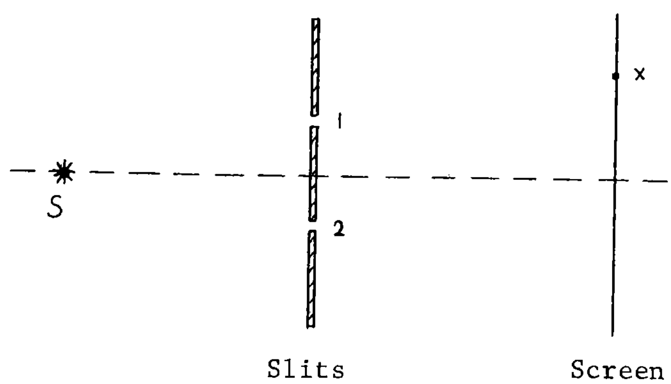
(Note that the average vertical spacing between the two routes is d/2). Show that the pattern is displaced upwards by $\frac{1}{2} T^2 \left(\frac{F}{M}\right)$, where $T = \frac{L}{v}$ is the classical time to go from slit to screen. Comment.

3-7. Electrons (spin 1/2) are emitted from a source S placed in front of a screen which contains two slits as shown. Assume that when an electron reaches a slit, it goes through with amplitude $\underline{\alpha}$ for "unflipped" spin and amplitude $\underline{\beta}$ for "flipped" spin. Assume further that it is impossible to distinguish which slit the electron passed through.

a) If all electrons are emitted with spin up, calculate the intensity distribution on the screen at point x in terms of α, β and amplitudes such as $\langle 1|s\rangle$ and $\langle x|1\rangle$.

b) How does this distribution differ from the case in which all electrons emitted are spin down and all other conditions are similar?

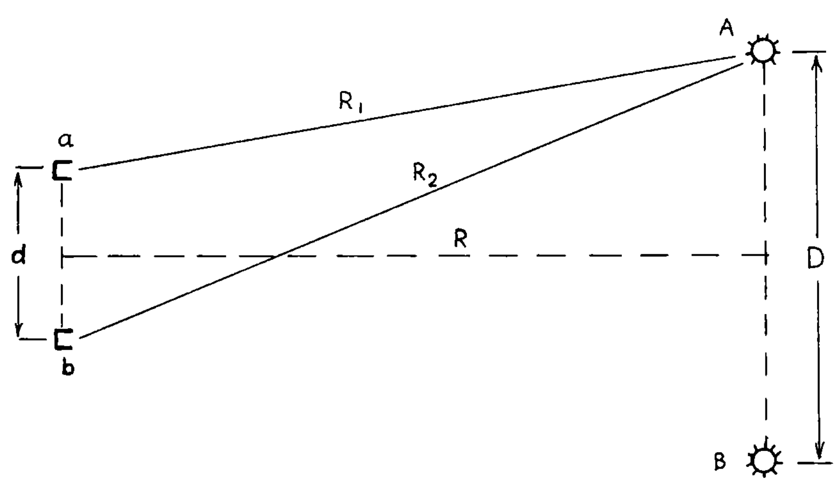
c) If the electrons are emitted with spins randomly up or down, again find how this case differs from part (a) assuming all other conditions unchanged.



3-8. Surprisingly, large interference effects can occur even when one of the interfering possibilities is not very probable. In the two hole diffraction experiment, if one hole is stopped down so that the probability of getting through is reduced by a factor of 100, show that the arrival probability at a maximum of the pattern is still 50 percent higher than at a minimum.

3-9. The diameter of the nearest stars is too small to be "seen" with the best telescopes (the angle subtended is less than the resolution of the telescope). The diameter of a star was first measured by Michelson using an optical interferometer. The method just barely works for the nearest stars. In 1956 Brown and Twiss (Nature 178, 1046, (1956)) proposed a new method, called "intensity correlation," for such measurements, and tested their method on the star Sirius. They took two parabolic reflectors (old searchlight mirrors) each with a photomultiplier tube at the focus. The outputs of the multipliers were fed by coax cables to a circuit that measured the average value of the product of the two currents (a so-called "correlator"). From the variation of this product with the separation of the two mirrors they determined the angle subtended by the star.

There were at the time many physicists who said that the method couldn't work. The argument was that since light came in photons which went either to one mirror or the other, there could be no correlation in the two currents. You can show that this argument is wrong by considering the following idealized experiment. There are two small sources say two light bulbs -- A and B, at a large distance from two photomultiplier tubes a and b with the geometry shown in the figure.



Counters are attached to the detectors a̲ and b̲ that measure the number of photons per second p_1 and p_2 arriving at each counter. The counters a̲ and b̲ are also connected to a "coincidence" circuit that measures p_{12} the counting rate for the <u>simultaneous</u> appearance (within some small time τ) of two photoelectrons.

Let $\langle a|A\rangle$ be the amplitude for a photon to arrive at a̲ from A in any particular resolving time interval. Then $\langle a|A\rangle$ is $c\ e^{i\alpha_1}$ where c̲ is a complex constant and α_1 is k̲ times the distance R_1 from A to a̲. Similarly $\langle b|A\rangle = c\ e^{i\alpha_2}$ with $\alpha_2 = k\ R_2$ where R_2 = the distance from A to b̲.

Show that the coincidence counting rate p_{12} is proportional to

$$2 + \cos 2k\ (R_2 \quad R_1)$$

How can this result be used to measure D if R is known? Ignore the fact that the actual process must be represented by a superposition of such models because light comes from all areas on the star's surface and not just from two points on the limbs.

CHAPTER 4

4-1. A broadcasting transmitter radiates 1,000 kilowatts at a frequency of 1.0 megacycle/sec.

a) What is the energy (in electron volts) of each quantum radiated?

b) How many quanta are emitted in each period of oscillation of the electromagnetic field? (The high coherence of these quanta is possible because they are Bose "particles".)

4-2. In a black body, $E(\omega)$, the <u>energy</u> per unit volume in the radiation with frequencies between ω and $\omega + \Delta\omega$, is given by Planck's Radiation Formula.

a) What is the behavior of $E(\omega)$ for small ω? For large ω?

b) At what frequency is there the most energy per unit frequency interval?

c) At what wavelength is there the most energy per unit wavelength interval?

d) Estimate the temperature of the sun by assuming that its maximum energy radiation occurs in the middle of the visible spectrum.

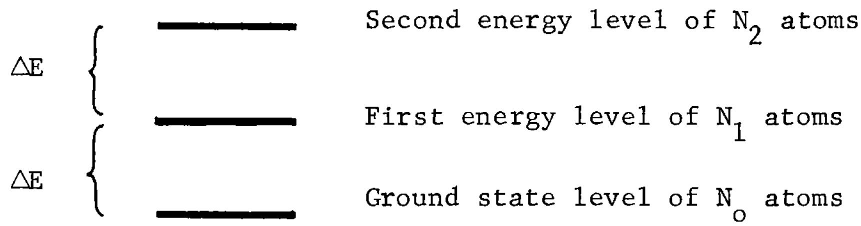
4-3. Estimate the strength of the magnetic field required to make the spins of the two electrons in a helium atom line up in the same direction. (Approximate the helium atom by a harmonic oscillator with a natural frequency corresponding to optical light. "Ground state helium" will have two electrons in the lowest level, with opposite spins. Because of the exclusion principle, one will have to go to the next level up if both are to have the same spin).

4-4. Before neutrons were discovered it was thought that nuclei contained protons and electrons. Show that the N^{14} atom (atom of nitrogen with nucleus of mass near 14 times the proton) would be a Bose particle.

Experiments (spectrum of the N_2 molecule) showed that this atom was a Fermi particle. This was the first evidence for a new nuclear particle. Show how the neutron hypothesis resolves the dilemma.

4-5. In a particular system, suppose "transitions" can occur between certain energy levels. That is, we assume the population or number of atoms in the energy levels is changed with an accompanying emission or absorption of quanta.

It is given that the two excited states and ground state are in thermal equilibrium with themselves when the total system is bathed in radiation of frequency $\hbar\omega = \Delta E$. Direct transitions with energy $2\Delta E$ are forbidden.



a) Solve for N_1/N_o and N_2/N_1 in terms of $n(\omega)$, the number of quanta.

b) Derive a simple relation for $n(\omega)$, the number of photons (bosons), involving $\Delta E/kT$ only.

c) Find approximate expressions for $n(\omega)$ in the limit,

1) $\hbar\omega \gg kT$

2) $\hbar\omega \ll kT$

4-6. In a laser a number of similar atoms are raised to an excited state. The presence of a small amount of some light of one kind then induces emission until all atoms contribute like an avalanche and thus create very many photons all of exactly the same wavelength and direction. Explain how the atoms can be "trained" to all emit in the same direction. Could you expect that some day one could develop a similar device for neutrinos (massless particles with spin 1/2)?

4-7. Show that if two non-identical particles do not interact, the probability that one goes from a to b while the second goes from c to d is the product of two factors P_{ab} and P_{cd}, where P_{ab} is the probability the first would go from a to b if the second were not present and P_{cd} is the probability the second would go from c to d if the first were not present. Is the restriction to non-identical particles essential?

4-8. Deuterium is a Bose particle which has spin one; thus a beam of deuterons can be in one of the three states +1, 0, -1. An experiment is performed in which deuterons are scattered from deuterons. What is the probability of detecting deuterons as a function of the scattering angle θ between the incident and target deuterons in the center of mass system. Assume there is no change of the spins during the scattering, and that $f(\theta)$ is the amplitude for a deflection θ.

4-9. Let $f_1(\theta)$ be the amplitude for the scattering of a π-meson from a proton and $f_2(\theta)$ be the amplitude for the scattering from a neutron. What would you guess for the probability that a π-meson would be scattered from a helium nucleus at the angle θ in terms of P_1 and P_2 the probabilities for scattering from protons and neutrons? Consider two cases:

a) That the recoil of the proton or neutron after the scattering breaks up the nucleus.

b) That the recoil is so small that the nucleus remains intact.

Can you say which case gives more scattering? Your answers depend upon the assumptions used in describing process (a).

4-10. A beam of neutrons is incident on a neutron target in a neutron scattering experiment. A detector is set up so as to detect neutrons which are scattered at an angle θ in the center of mass system.

There is an amplitude f for a particle in the incident beam to be scattered into the detector without any change in spin. There is also an amplitude g for a particle in the beam to scatter into the detector with a spin flip (interchanging its spin direction with the particle from the target.) If one assumes that f and g are independent of θ, what probability for a count in the detector would one predict, if:

a) The incident and target neutrons both have spins in the +z direction.

b) The beam of neutrons have their spins aligned in the +z direction and the target neutrons have spins aligned in the -z direction.

c) The incident beam is unpolarized while the target neutrons are polarized in the +z direction.

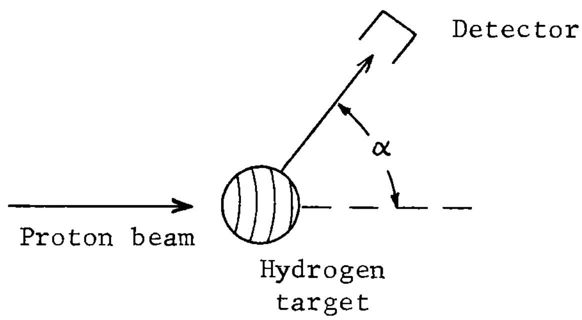
d) Both the incident and target neutrons are unpolarized.

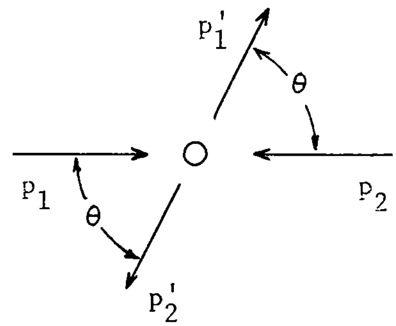
e) What would the answer to part (a) be if the target was polarized protons and the scattering amplitudes for neutron-proton scattering equaled those for neutron-neutron scattering? Assume the detector has the same efficiency for detecting neutrons and protons.

4-11. A beam of non-relativistic protons is passed through a thin target of liquid hydrogen, and as shown in the figure, scattered protons are counted at some angle α with respect to the incident beam direction. The scattering can be analyzed in the center-of-mass system as shown in the figure. Two protons (p_1 and p_2) approach with equal velocities; after the collision two protons (p_1 and p_2') leave the collision at the angle θ. If we define the "z"-axis as perpendicular to the scattering plane, each proton can have J_z(z-component of spin angular momentum) either $\pm \hbar/2$. We say the spin is either "up" or "down". Suppose both protons have spin "up", and that the amplitude that p_1 is scattered into a detector at θ

is $f(\theta)$. Since we cannot tell which proton is detected, the amplitude that a proton will appear at the angle θ is $f(\theta) \quad f(\pi - \theta)$. The minus sign appears because protons are "fermions". We can then say that the probability that a proton is observed in the detector is

$$|f(\theta) \quad f(\pi \quad \theta)|^2$$





Suppose now that p_1 has spin "up" and that p_2 has spin "down," and that the amplitude that p_1 is scattered into the detector with no change of spin is $f'(\theta)$ and that the amplitude for scattering with a spin flip is $g(\theta)$ because the scattering amplitude depends on the relative spin orientations.

In this case the amplitude that an "up" proton will arrive at the detector can be written $f'(\theta) + g(\pi \quad \theta)$.

a) What is the relation between θ and α?

b) What is the amplitude for the "up"   "down" case that a proton will be scattered into the detector with spin "down"?

c) Suppose that a "natural" beam of unpolarized protons is scattered from a "natural" (unpolarized) target, and that the detector cannot distinguish between the two polarizations. What is the scattering probability for the angle θ?

d) Show that if $f' = f$ and $g = 0$, the scattering of protons of random spins is equivalent to a mixture of "pure fermion" scattering for which the amplitude is $f(\theta) \quad f(\pi - \theta)$ and "pure boson" scattering for which the amplitude is $f(\theta) + f(\pi - \theta)$; that is,

$$P = A\ \left|f(\theta) \quad f(\pi - \theta)\right|^2 + B\ \left|f(\theta) + f(\pi - \theta)\right|^2$$

Find A and B.

4-12. Suppose N electrons are in a very large box of volume V in a condition to give the least possible energy. If we disregard the interaction between the electrons show that each mode of the box is occupied by just two electrons, provided that the momentum of the mode ($\hbar k = p$) has a magnitude less than p_{max} where

$$N = \int_0^{p_{max}} V \quad 2 \quad 4\pi p^2 dp/(2\pi\hbar)^3$$

What is the energy U of all the electrons? Express this internal energy U in terms of the volume of the box, and find thereby the pressure exerted by this so-called "degenerate electron" gas. Show the pressure-volume reaction is of the form PV^γ = constant, and find γ.

4-13. Matter in white dwarf stars is so highly compressed that the theory of the last problem applies to them. If ρ is the density of the material, $\rho/2M_p$ is the number of protons per cubic meter (where M_p is the mass of a proton and we suppose the nuclei have about as many neutrons as protons), so set $N/V = \rho/2M_p$ in the equations of problem 12. The equations of equilibrium of a star of such material held together by gravitation appear in a book on astrophysics as

$$P = A\rho^{5/3}$$

$$dP/dr = -G\rho M(r)/r^2$$

$$dM(r)/dr = 4\pi\rho r^2$$

Can you explain the reason for these equations and supply a formula, or a numerical value, for the constant A? Suppose all pressure is exerted by the degenerate electrons, the nuclei having virtually no effect (why?).

CHAPTER 5

5-1. Prove the statement made in Chapter 5 that if the apparatus C can be broken into two parts, A and B, then

$$\langle \chi | C | \varphi \rangle = \sum_k \langle \chi | B | k \rangle \langle k | A | \varphi \rangle$$

5-2. Three "improved" Stern-Gerlach apparatuses, described in Chapter 5, which separate a beam according to the value of the z-component of spin but which bring the beam together spatially, are put in series with each other, and a beam of spin one particles is sent through. The first and third are lined up with the same orientation while the middle is set at an arbitrary angle. In the notation of Chapter 5, this would appear as:

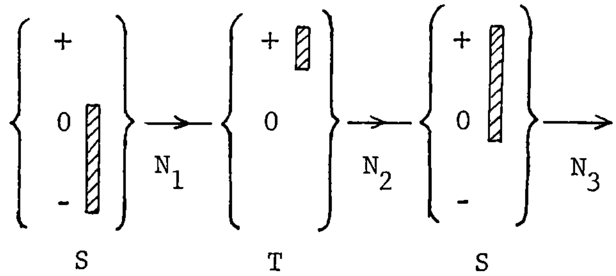
$$\begin{array}{ccccc} \left\{\begin{matrix} + \\ 0 \\ \; \end{matrix}\right\} & \quad & \left\{\begin{matrix} + \\ 0 \\ \; \end{matrix}\right\} & \quad & \left\{\begin{matrix} + \\ 0 \\ \; \end{matrix}\right\} \\ S & & T & & S \end{array}$$

a) If one slot of T is "open," does the proportion of the beam found in each of the three states of the final S system depend on the input state, i.e., on the proportion of the beam in +S, 0S, or -S. Why?

b) What if two slots of T are "open"?

c) What if three slots of T are "open"?

5-3. A set of three "improved-type" Stern-Gerlach experiments is set up for spin one particles as follows:



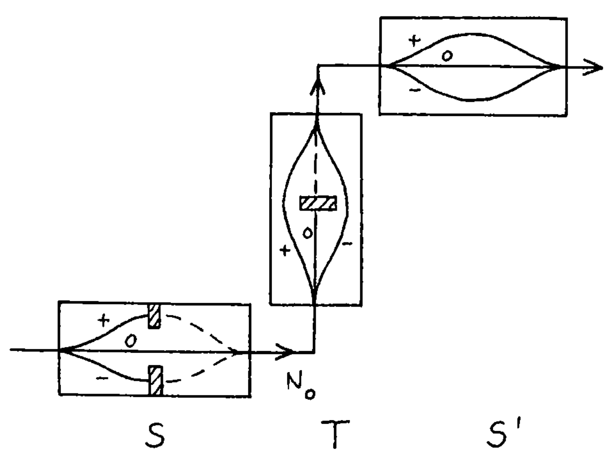
The three apparatuses are placed along a straight line, but the T apparatus is rotated about this line by an angle of 90^o with respect to the two S apparatuses. A beam of spin one particles enters from the left. The beam which leaves the first S apparatus has an intensity of N_1 particles per second.

a) What is N_2, the intensity of the beam leaving the T apparatus?

b) What is the intensity N_3 of the beam that leaves the last S apparatus?

c) What are N_2 and N_3 if all the "stops" are removed from the T apparatus?

5-4. Consider a sequence of modified Stern-Gerlach apparatuses S, T, S' used with particles of spin 1: (T is rotated by 90^o with respect to S and S').



For N_o particles coming out of the S apparatus, find the expected number of particles emerging from S' in the states $|+S'\rangle$ and $|OS'\rangle$. (Call them $N_{+S'}$ and $N_{OS'}$.)

Suppose we had available "transparent detectors," which could be placed into the + and beams of the T apparatus. Let these detectors have the property that when a particle passes through one of them a signal is generated, without the spin state of the particle being changed. Furthermore the momentum of the particle is not appreciably changed, in the sense that its trajectory through the T apparatus can be considered to remain the same as if the detector were not present.

With detectors present in the + and states of T (the 0 state remains blocked), what is the probable number of counts recorded for N_{+T} and N_{-T}, and for $N_{+S'}$ and $N_{0S'}$, if a total of N_o particles emerge from the S apparatus? How would the answer for $N_{+S'}$ in the experiment described above change, if it was discovered after the experiment that no counts had been recorded for N_{+T} and N_{-T} because the signals from the detectors had not been recorded? If each detector is only 50% efficient (i.e., 50% of the time there is no interaction between the particle and the detector) what is the answer for $N_{+S'}$ and $N_{0S'}$? If the blocks from the + and states of S are removed and N_o particles are sent into S, what are $N_{+S'}$ and $N_{0S'}$? (The detectors in T are removed also.) Assume the incident beam is unpolarized.

6-1. Imagine that atoms with a spin of 1/2 are filtered by a sequence of two "improved" Stern-Gerlach apparatuses. It is supposed that each apparatus is arranged to permit only one beam to go through, as indicated in figure A. For each of the arrangements shown in figure B, consider that N unpolarized atoms enter at P. Give the expected number of atoms to arrive at Q.

Fig. A

Side View

Top View

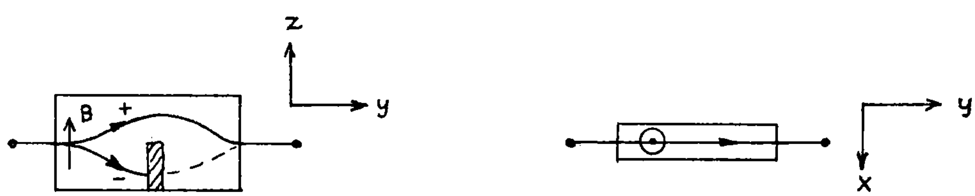
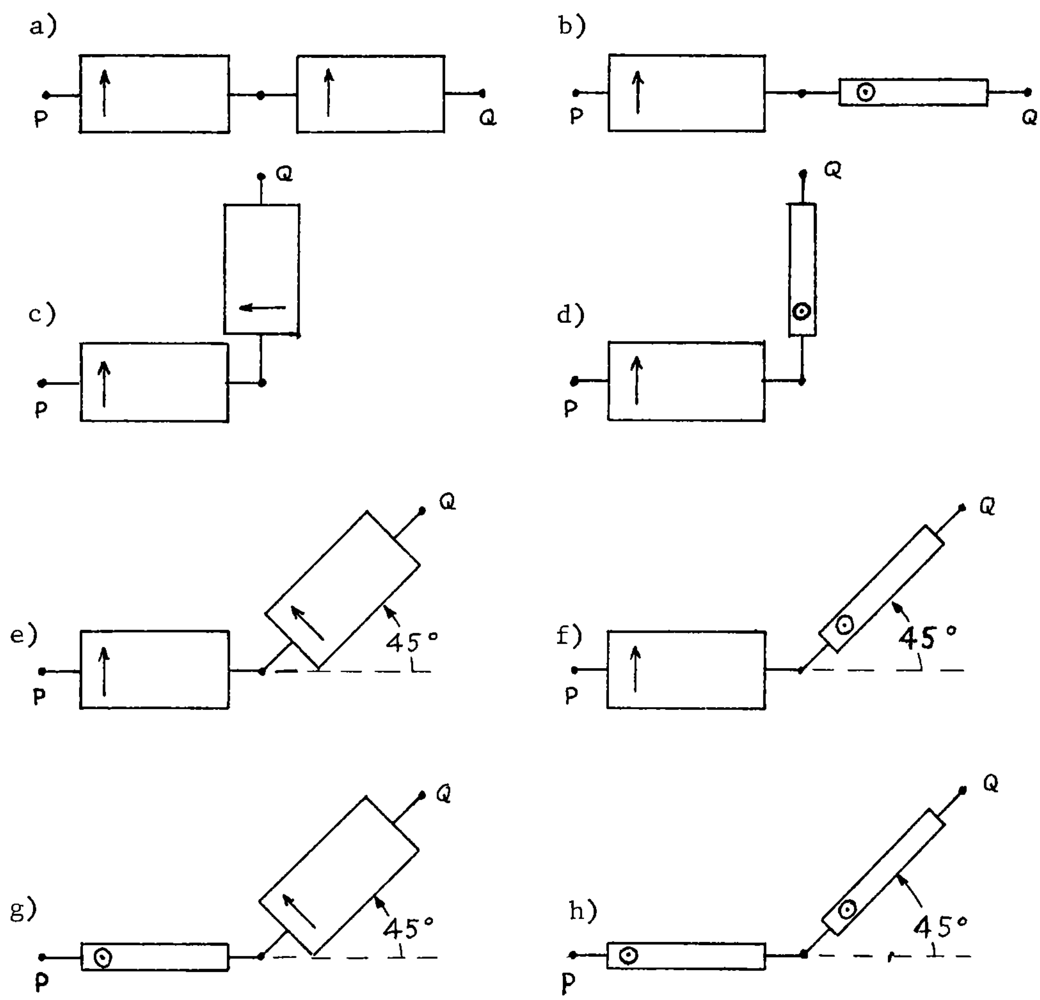


Fig. B



6-2. A spin one-half particle enters some apparatus with amplitudes a and b to have spin up and down along the z-axis. Show that the probability for it to arrive at any given point in the apparatus is necessarily of the form $|aX + bY|^2$ where X and Y are some complex numbers characteristic of the apparatus. In terms of X, Y what is the probability to arrive if:

a) the incoming particle has spin up in z? Down?

b) the incoming particle has spin up along the x axis? Down?

c) the incoming particle has spin up along axis with polar angles θ, ϕ ?

There are several ways to imagine that the incoming particles are in "random" spin states:

A) For some experiments the electrons are up along z, for others down, a coin being flipped each time to decide which.

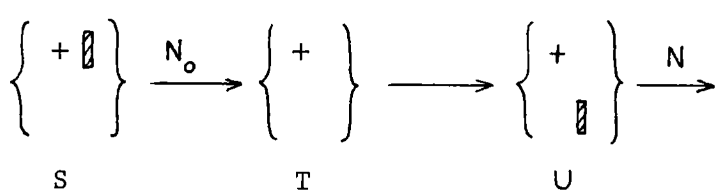
B) The same as (A, except they are either up or down along x.

C) Each is oriented in some direction θ, ϕ but all directions are chosen at random (so we average over solid angle $\sin\theta \, d\theta \, d\phi/4\pi$).

Find the average probability to arrive in the apparatus for each of the above kinds of "random" circumstances (A), (B), (C), and show they are equal.

Suppose spin 1/2 particles are coming out of a hole, being prepared by one of the methods (A), (B), (C) on the other side. Can you think of any way at all that, by observations on your side of the hole, you could tell whether method (A), (B) or (C) is actually being used?

6-3. Fig. 1



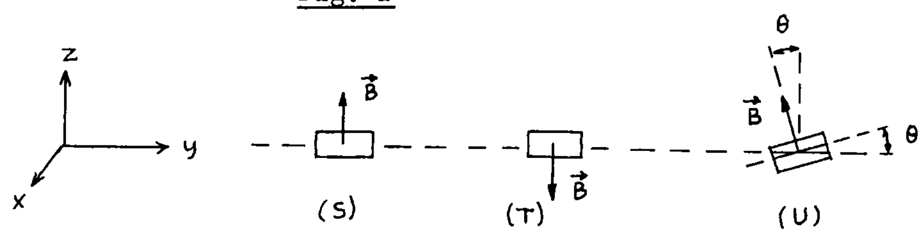
Three modified spin 1/2 Stern-Gerlach apparatuses S, T and U are in series as shown. Express N, the number of spin 1/2 particles coming out of U, in terms of N_o, the number coming out of S, and in terms of objects like $\langle +T|+U\rangle$, etc.

Now consider the same apparatus as above, but with the B-field orientations given in figure 2. In particular,

i) The B-field of the T-apparatus is rotated anti-parallel to the B-field of the S-apparatus.

ii) The B-field of the U-apparatus is rotated an angle θ from the z-axis.

Fig. 2



a) Find $\langle +T|-S\rangle$ and $\langle -T|-S\rangle$ explicitly.

b) Find explicitly $\langle +U|-S\rangle$ using, as an exercise, only the transformation table for rotations around the z and the y axes.

c) Find limiting forms of your answer in (b) for

i) $\theta = 0^{\circ}$

ii) $\theta = \pi$.

Explain your answer for $\theta = \pi$ in (c) upon comparison with $\langle +T|-S\rangle$ of (a).

6-4. A piece of calcite splits a beam of light (going in the z direction) into two, corresponding to x and y. A single photon entering a calcite S has an amplitude to be in one or the other of these two beams, x, y. A similar calcite (turned backwards) can be used to bring the two beams together again, etc., just as in the Stern Gerlach apparatus. Another calcite T can have its axis tilted by angle θ in the x-y plane, splitting into states x', y', or xT, yT. Find the amplitudes $\langle xT|xS\rangle$, $\langle yT|yS\rangle$, etc., from your knowledge of the classical theory of polarized light by making sure that in each case the intensity in a beam of a large number of photons will agree with the classical result. Consider rotations only about the propagation axis z, for light cannot be brought to rest (rotations about the other axes can be described by their effect on the direction of propagation, rather than on the polarization in this way light, which is a two state system is very different in its transformation properties from an electron, which is also a two state system.)

6-5. Find all four of the elements of the matrix $\langle j|A|i\rangle$ where i and j are x and y for the following pieces of apparatus through which light may be passed:

a) A x, y calcite splitter and restorer with beam y blocked.

b) A calcite splitter and restorer set at angle θ with beam y blocked.

c) Polaroid set at axis x to pass.

d) Polaroid set at axis θ in x-y plane to pass.

e) A xy calcite splitter and restorer with a block of glass in beam x which delays the phase of that beam by angle $\emptyset$.

f) A xy calcite splitter and restorer with the same glass in both beams.

g) A calcite splitter and restorer set at 45° with a glass in the x beam delaying the phase by 90°

h) A quarter-wave plate.

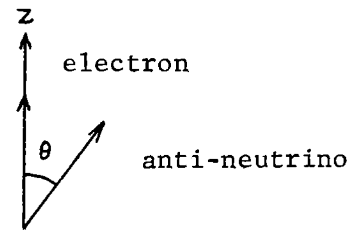
i) A birefringent material, at axis x (give general formula in terms of thickness).

j) A sugar solution which turns the plane of polarization to the right by angle θ.

k) A device which splits the beam into x, y beams, changes the x beam to the y direction (by putting it through sugar water turning its polarization by 90°) and puts the two beams back together again.

ℓ) Show that you can make perpetual motion with the device in part (k) above.

6-6. According to the theory of β-decay in a certain kind of nuclear decay (in which the nucleus suffers no change of angular momentum or parity, called "Fermi allowed") an electron moving along the z axis with velocity v̲ is emitted with amplitude $\sqrt{1 - v/c}\ \sin\theta/2$ with spin up along z, and with amplitude $\sqrt{1 + v/c}\ \cos\theta/2$ with spin down along z. (Here θ is the angle, from z, that the anti-neutrino is emitted. Incidentally, anti-neutrinos always have their spin along their direction of motion).



a) What is the probability that the spin is up along z? Down?

b) What is the probability that the spin is in the direction* +x (if the neutrino is in the xz plane)? The direction -x?

c) What is the probability in the directions* ±y?

d) If, as is usual, the neutrino is not observed (average over all anti-neutrino directions) what is the answer to (a)?

Note: Strictly speaking, this refers to the coordinate system moving with the electron. Just use regular formulas for combining amplitudes.

6-7. In the last problem, making a Lorentz transformation of velocity v̲ along z to bring the electron to rest does not change the numerical values of the amplitudes to have spin up and down along this axis. (Can you think of a reason why this might be so?) However, the apparent direction the anti-neutrino is, of course, altered. Show that the amplitudes in the above problem mean that the electron spin is lined up opposite to the direction (and hence opposite to the spin) of the anti-neutrino in the system in which the electron is at rest. (This occurs because the nucleus lost no angular momentum.)

CHAPTER 7

7-1. A particle of spin one in a magnetic field in the z direction has three states (labeled +, 0, -) with energies $+\mu B$, 0 and $-\mu B$ respectively. Show quantum mechanically that in an inhomogeneous magnetic field a beam of such particles would be split into three beams and find the laws giving the deflection, assumed small (in terms of the length of the field, initial momentum of the particles, etc.) Next, show quantum mechanically that such a particle will "precess" (use the coefficients of Sec. 5-7, to make an argument like that of Sec. 7-5.) Suggest at least two independent ways in which μ might be measured experimentally.

8-1. A beam of spin one-half particles with magnetic moment μ is sent into a Stern-Gerlach filter which passes only particles in the $|+\rangle$ state (spin up) with respect to the z-axis. The particles then spend the time T in a uniform magnetic field B_o which is parallel to the x-axis. After leaving the uniform field the particles enter a second Stern-Gerlach filter which passes only particles in the $|-\rangle$ state (spin down) with respect to the z-axis. Assume that $\vec{\mu}$ and $\vec{J}$ are parallel.

a) What is the smallest value of B_o for which all of the particles will get through the second filter?

b) If the particles spend only half as long in the same field, what is the probability that they will get through the second filter?

8-2. A beam of spin one-half particles with magnetic moment μ is sent into a Stern-Gerlach filter which passes only particles in the $|+\rangle$ state (spin up) with respect to the z axis. The beam then goes into a magnetic field which is at 45^o with respect to the z-axis in the x-z plane. At a time T later, what is the probability that the particles will be found with $J_x = \hbar/2$ or with $J_y = \hbar/2$? Again assume $\vec{\mu}$ and $\vec{J}$ are parallel.

8-3. A spin 1/2 particle has its spin pointing in the +z direction at time t = 0. The particle is located in an apparatus such that the amplitude per unit time to go from the plus to minus z-state is the same as the amplitude to go from minus to plus z-state and both are equal to i times some positive constant $(A/\hbar)$, i.e., $H_{12} = H_{21} = -A$. Further H_{11} equals H_{22} and may be taken equal to zero.

a) What is the probability for finding the particle in the +z state at time T?

b) Find the two combinations of amplitudes to be in the

+z and -z states which correspond to stationary states. What are the energies of these states?

c) At any time T, there exists an axis along which the probability of spin up is unity. In what direction is this axis?

d) Can you think of a physical piece of equipment that would have this effect?

CHAPTER 9

9-1. In Chapter 9, the probability of an ammonia molecule making a transition from state $|II\rangle$ to state $|I\rangle$ by shining microwaves on the molecule was calculated; state $|II\rangle$ has a lower energy than state $|I\rangle$ so this corresponds to the absorption of energy from the radiation.

Redevelop these ideas to find the probability for inducing emission by the ammonia molecule. How does the probability of absorption compare to that of emission? How is this probability related to the Einstein A and B coefficients defined in Chapter I-42? Find the rate of spontaneous emission by the ammonia molecule.

9-2. Protons (magnetic moment μ) in a water sample are exposed to a uniform magnetic field. Now although the magnitude of the field B is constant, the field changes its direction in time for a nuclear magnetic resonance (NMR) experiment:

$$B_x = B \sin\theta \cos\omega t$$
$$B_y = -B \sin\theta \sin\omega t$$
$$B_z = B \cos\theta$$

Initially ($t = 0$) we are given that the spin of the protons are along the magnetic field in the + 1/2 state. Assume θ, the angle from the z-axis in spherical coordinates, is <u>very small</u>. What value must ω have for resonance? What is the probability for the particle at time t to have a spin <u>down</u> along the z-axis, for ω at resonance?

CHAPTER 10

10-1. A spin 1/2 particle is placed in a large magnetic field B_o. An oscillating magnetic field $2B_n \cos \omega t$ whose magnitude is much smaller than B_o is applied in a direction normal to B_o. If the spins of the particle were initially lined up in a direction opposite to B_o, what is the probability after a time T of having the spin lined parallel to B_o?

CHAPTER 11

11-1. Show that the Pauli spin matrices can be treated as the components of a vector $\vec{\sigma}$ which obeys the following rules:

$$\vec{\sigma} \times \vec{\sigma} = 2\, i\, \vec{\sigma}$$

$$\vec{\sigma} \quad \vec{\sigma} = 3 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Find $\sigma_x\ \sigma_y\ \sigma_z$.

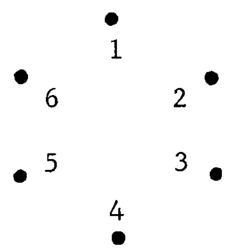
11-2. Carbon dioxide is a linear molecule (OCO) which likes to pick up an extra electron and become a negative ion. Imagine that the electron would have the energy E_o if it were attached to either oxygen atom, or the energy E_c if it were attached to the carbon atom. The stationary states need, however, have neither of these energies, because there is some small chance for the electron to jump between an oxygen atom and the carbon atom. (Assume that the chance to jump directly from oxygen to oxygen is negligible.)

a) Find the possible energy levels of the CO_2 ion in terms of E_o, E_c, and one other parameter.

b) Give a physical description of each of the stationary states, in the case that the energies E_o and E_c are equal.

11-3. In the methane molecule, four hydrogen atoms are placed in the four corners of a tetrahedron with a single carbon atom in the center. In the methane ion, an electron is missing from one of the four bonds, thus leaving a "hole" which can "jump" from any of the H atoms to another. This is an example of a four state system. By using symmetry arguments to reduce the number of different Hamiltonian matrix elements to a minimum, predict the number of different energy levels you would expect to observe from the electronic structure of the methane ion. Neglect the rotational and vibrational interactions of the atoms. Express the separation of the levels in terms of the fewest possible matrix elements you can.

11-4. Consider six atoms spaced equally around a ring, as shown:



Add one extra electron, and define base states $|1\rangle$, $|2\rangle$, $|6\rangle$, where $|1\rangle$ means the electron is at atom 1, $|2\rangle$ means it is at atom 2, etc. Assume that the extra electron has a definite amplitude to jump from any atom to either one of the two nearest neighbors, but no amplitude to jump further.

Show that $|I\rangle$ is a stationary state if the amplitudes $C_i = \langle i|I\rangle$, with $|i\rangle$ being the i^{th} base state, are all equal to $(1/\sqrt{6}) \exp (- \frac{i}{\hbar} E_I t)$. Find E_I. How many other stationary states are there?

It can be shown that if ψ is a stationary state the amplitudes $C_i = \langle i|\psi\rangle$ are related in the following way, provided δ is suitably chosen:

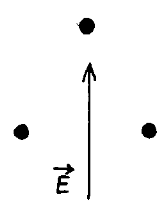
$$C_2 = C_1 e^{i\delta}$$
$$C_3 = C_2 e^{i\delta}$$
$$C_4 = C_3 e^{i\delta}$$
$$C_5 = C_4 e^{i\delta}$$
$$C_6 = C_5 e^{i\delta}$$

What are the allowed values of δ? Find the energy level diagram for the system and give the spacings between levels.

11-5. A molecule is made up of three similar atoms placed in the corners of an equilateral triangle. In the negative ion of this molecule, an additional electron is added which is able to jump from any of the three atoms to another.

a) Take the Hamiltonian matrix element to jump from either atom to another to equal -A and calculate the energy spacings of the molecular ion.

b) An electric field is applied to the ion in the plane of the ion and pointing to one apex as shown below. If the strength of the field is such that the potential energy of the electron at this apex is increased by $\epsilon A = 0.01$ A above the potential energy at the other corners, how much and in what way are the energy spacings changed?



CHAPTER 12

12-1. Calculate the amount of splitting in the j = 1 level of a hydrogen atom which is placed in interstellar space where the magnetic field is of the order of 10^{-5} gauss, at the surface of the earth where the magnetic field is about 1/2 gauss and in the largest magnetic fields which have been produced in the laboratory, i.e., fields of about 100,000 gauss. Express your answers both in frequency and wavelength.

CHAPTER 13

13-1. Consider an infinite line of atoms with equal spacing $\underline{b}$, and assume that an electron can be attached to any given atom in two configurations i and j with different energies E_i and E_j, i.e., assume that a suitable set of base states are:

$$\left|\begin{array}{l}\text{electron at atom } x_n \\ \text{in configuration i}\end{array}\right\rangle = |x_n, i\rangle$$

$$\left|\begin{array}{l}\text{electron at atom } x_n \\ \text{in configuration j}\end{array}\right\rangle = |x_n, j\rangle$$

Assume further that the elecrron can jump from one atom to its nearest neighbor with amplitudes:

$$-\frac{A_{ii}}{i\hbar} \quad \text{to go from} \quad |x_n, i\rangle \text{ to } |x_{n+1}, i\rangle \quad \text{or} \quad |x_{n-1}, i\rangle$$

$$-\frac{A_{jj}}{i\hbar} \quad \text{to go from} \quad |x_n, j\rangle \text{ to } |x_{n+1}, j\rangle \quad \text{or} \quad |x_{n-1}, j\rangle$$

$$-\frac{A_{ji}}{i\hbar} \quad \text{to go from} \quad |x_n, i\rangle \text{ to } |x_{n+1}, j\rangle \quad \text{or} \quad |x_{n-1}, j\rangle$$

$$-\frac{A_{ij}}{i\hbar} \quad \text{to go from} \quad |x_n, j\rangle \text{ to } |x_{n+1}, i\rangle \quad \text{or} \quad |x_{n-1}, i\rangle$$

Consider the case when $A_{ij} = A_{ji} = -A$ and $A_{jj} = A_{ii} = -B$.
Follow the procedure described in Chapter 13 to find the allowed values of the energy of such a system. Describe the band structure in the case that $|E_i - E_j| \ll 2B$ and $|E_i - E_j| \gg 2B$. Check your answer with the solution found in Chapter 13.

13-2. Consider an infinite line of atoms composed of two kinds of atoms, "a" types and "b" types as shown:

a	b	a	b	a	b	a	b	a
n-2	n-2	n-1	n-1	n	n	n+1	n+1	n+2

Let the amplitude for an electron to be found on the n'th "a" type, be C_n^a while the amplitude to be found on the n'th "b" type be C_n^b. Assume that the energy of an electron on an "a" atom is $E_o + \Delta E$ while the energy of an electron on an "b" atom is $E_o - \Delta E$: further assume that the Hamiltonian matrix elements equal -A for jumps to nearest neighbors. The spacing between atoms is b.

Calculate and plot roughly the energy of a stationary state as a function of k. (You will have two energies for a given value of k.) What limits can you put on the electron's wave number k, in order to include every state exactly once?

13-3. Scattering from an impurity: Referring to the example in Chapter 13, let the atom at n = 0 be different in a different way. Let $H_{00} = E_o$, $H_{01} = H_{10} = H_{0(-1)} = H_{(-1)0} = -B$, where $B \neq A$. Find f and g, and veryify that $|f|^2 + |1 + g|^2 = 1$.

13-4. For both the preceding problem and for the example in Chapter 13, f = g. It is easy to verify that f = g is also true in the general case, which combines the two.

"Conservation of particles" therefore gives, for the general scattering in one dimension:

$$|f|^2 + |1 + f|^2 = 1$$

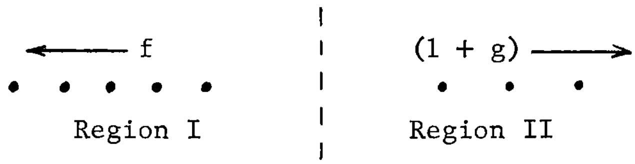
a) Show that this requires that $\mathrm{Re}\left(\frac{f}{1 + f}\right) = 0$

b) Show that f may be written:

$$f = ie^{i\eta} \sin \eta, \text{ where } \eta \text{ is real.}$$

The quantity η is called the "scattering phase shift" and tells both the phase and magnitude of the scattered wave. (True in the three-dimensional as well as in the one-dimensional case.)

13-5. Consider the one-dimensional analog of an interface where an infinite crystal changes its properties:



Let the particles be incident from the left, as in Chapter 13.

In region I we have the parameters E_o, $-A$, b, and in region II E_o', $-A'$, and b'. The amplitude analogous to A and A' which applies between the two atoms on either side of the interface is B. (Assume A, A', B, are all real.)

a) Show that $(1 + g) = \frac{B}{A'}(1 + f)$ at the interface between atoms $n = 0$ and $n = +1$.

b) Find f in terms of A, A', B, kb, $k'b'$.

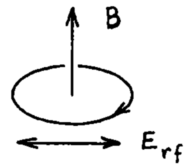
Show that if $(k'b')$ is imaginary $|f| = 1$. What does this mean physically? (What values of $(E - E_o')$ produce complete reflection?)

c) Verify conservation of particles by showing that

$$|f|^2 + |1 + g|^2 \frac{(v_g'/b')}{(v_g/b)} = 1 \qquad ,$$

where v_g and v_g are the group velocities in the two regions. Can you explain the factor multiplying $|1 + g|^2$?

14-1. Cyclotron resonance experiments are usually performed as shown schematically below:



$B = B_o$, a static magnetic field in the z-direction

$E_{rf} = E_o \cos \omega t$, along the x-axis.

The resonance at ω_c, the cyclotron resonance frequency, is detected by a change in the power absorbed from the E_{rf} field. From elementary considerations of a particle orbit in a uniform magnetic field,

$$\omega_c = \frac{qB}{m^*} \quad , \quad \text{where } m^* \text{ is the}$$

effective mass. Assume throughout that m^* does not depend on the direction of the particle motion.

For the equation of motion of an electron (or hole) in a semiconductor, take:

$$m^* \left(\frac{d\vec{v}}{dt} + \frac{1}{\tau} \vec{v} \right) = q(\vec{E} + \vec{v} \times \vec{B}) \quad , \quad \text{where } \tau \text{ is}$$

the mean time between collisions. (See Sections 32-1 and 32-6 of Vol. II.) Let $v_x = v_o e^{i\omega t}$ and $E_x = E_o e^{i\omega t}$, and show that

$$\frac{v_x}{E_x} = \frac{q\tau}{m^*} \left[\frac{1 + i\omega\tau}{1 + (\omega_c^2 - \omega^2)\tau^2 + 2i\omega\tau} \right]$$

Power absorption is proportional to Re $[v_x/E_x]$. Why? Describe how both τ and m^* can be obtained from cyclotron resonance data. Note that detection of a resonance requires $\omega_c \tau > 1$. What does this signify physically?

14-2. The energy diagram for holes in a typical "p-n" junction (as in a diode) is shown in Fig. a if no external voltage is applied. In equilibrium there is a "thermal generation" hole current I_g of holes which diffuse from the n to the p region and just equals a "recombination" current I_r of holes which goes from the p to the n region.

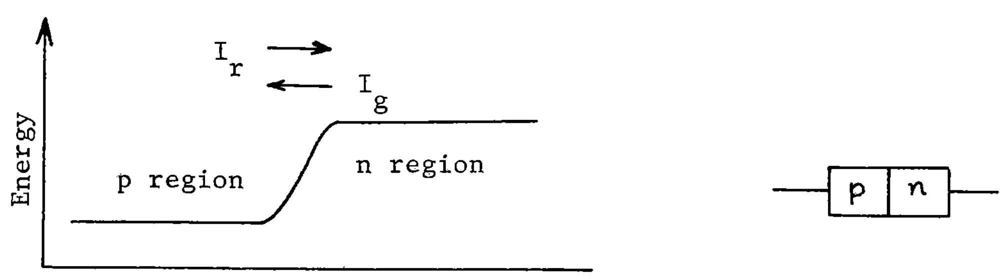
If a "reverse voltage bias" is applied, the energy diagram is as in Fig. b and if a "forward bias" is applied it changes to that shown in Fig. c.

By considering the currents first in equilibrium and then with bias voltages, show that the net hole current has the form

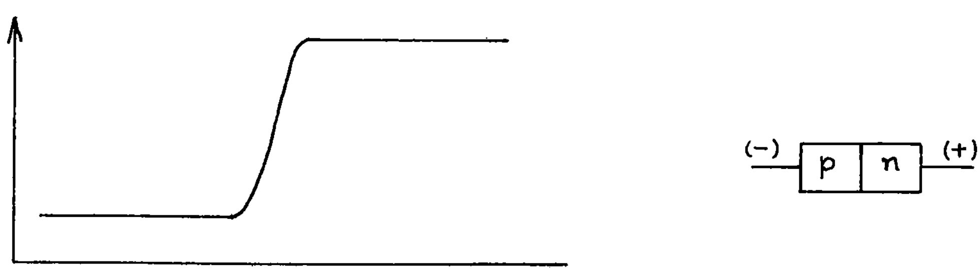
$$I(\text{holes}) = I_g(e^{qV/kT} \quad 1)$$

What is the relationship for the total current? (V is the voltage applied to the junction.)

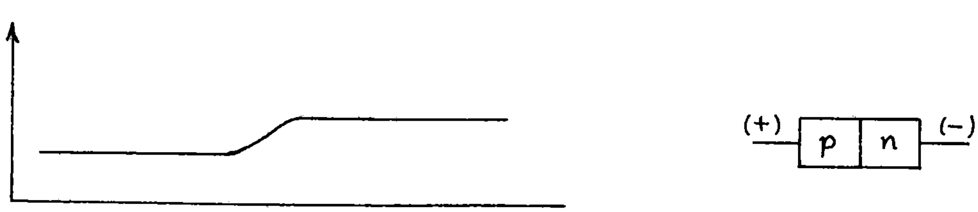
a)



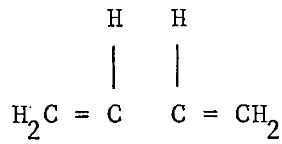
b)



c)



15-1. The structure of the butadiene molecule can be represented as

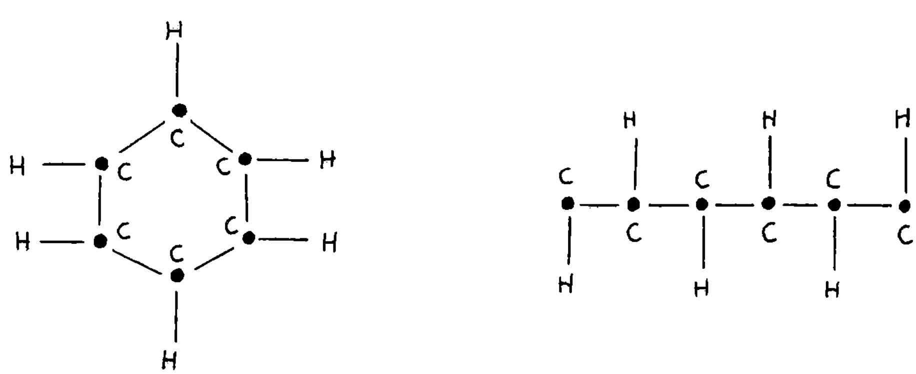


If we remove (theoretically) the four electrons of the double bonds, and then add them separately we can treat the problem in terms of the independent particle model.

Specifically, we can consider a four pit system with the usual energies E_o and Hamiltonian elements $-A$. What wavelength radiation is emitted when butadiene molecules make transitions from the first excited to the ground state? Assume $A = 1$ ev.

In singly ionized butadiene, only three of the electrons of the double bonds are present. What can you say about how these electrons are distributed in the molecule?

15-2. Estimate the energy needed to break a benzene ring by using molecular orbital theory (in the independent particle approximation) to calculate the energy difference between configurations (a) and (b) shown below. Find the answer in electron volts per molecule by utilizing the fact that the transition from the first excited state of benzene to the ground state produces radiation of wavelength about 2,000 Angstroms.



(a) (b)

15-3. A ferromagnetic material at very low temperatures can be discussed in terms similar to the spin waves discussed in Chapter 15. Specifically, for any mode K with an energy $E_K \approx K^2 b^2 A$, there is a probability distribution, based on thermodynamics, for finding either none, one, two, three, etc., down spins in a ferromagnet which at zero temperature has all the spins aligned up. Show that the mean number of atoms with down spins is proportional to

$$1/(e^{E_K/kt} \quad 1)$$

If these ideas are extended to three dimensions, $E_K \approx Ab^2(K_x{}^2 + K_y{}^2 + K_z{}^2)$ and the total number of down spins/unit volume is given by

$$\frac{\text{Number of down spins}}{\text{vol}} = \int \frac{d^3K/(2\pi)^3}{e^{E_K/kt} \quad 1}$$

Show why this is so.

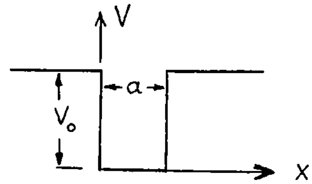
In the limit as T goes to zero, the magnetization goes to the saturation magnetization, M_{sat}. Show that at low temperatures the ratio of the magnetization to the saturation magnetization can be written as

$$\frac{M}{M_{sat}} = 1 \quad \text{const } T^{3/2}$$

$$= 1 - \left(\frac{kT}{4\pi A}\right)^{3/2} \left[\frac{4}{\sqrt{\pi}} \int_0^\infty \frac{x^2 dx}{e^{x^2} \quad 1} \right]$$

Evaluate the integral by expanding the integrand in a series.

16-1. Consider in one dimension the motion of a particle of mass m bound in a square well potential like:



$$V = V_o \qquad x < 0 \quad \text{or} \quad x > a$$

$$V = 0 \quad , \quad 0 < x < a$$

For simplicity, let $V_o \to \infty$.

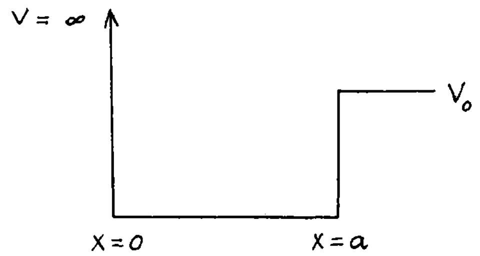
a) For the stationary state of lowest energy, E_o, $\psi_o(x,t) = u_o(x)\ e^{-iE_o t/\hbar}$ We must have $u_o(x) = 0$ just outside the well (i.e., at $x = \ \epsilon$ or $x = a + \epsilon$). Why?

b) Solve the Schrödinger Equation inside the well, subject to the condition stated in (a). Sketch $u_o(x)$ and find E_o. ($u_o(x)$ need not be normalized.)

c) Find the energy difference between the lowest state and the first excited state.

d) For the lowest state, roughly sketch the probability distribution for finding the particle with momentum p in the range dp. No exact integrations are required. Don't worry about normalization. But, be sure to indicate the scale along the momentum axis.

16-2. Consider the motion of a particle of mass m in a one dimensional potential well defined by:



$V = \infty \qquad x < 0$

$V = 0 \;, \quad 0 < x < a$

$V = V_o \;, \quad x > a$

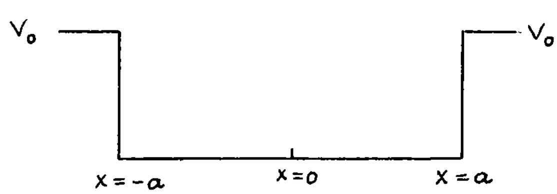
a) Find the value of V_o such that the energy of the particle in the ground state for this well differs by 10% from the ground state energy for a well with $V_o \to \infty$.

b) Let V_o be equal to the value found for part a), for the well defined by:

$$V = V_o \qquad x < -a$$

$$V = 0 \qquad a < x < -a$$

$$V = V_o \qquad x > a$$



Without detailed calculation, give the energy of the first excited state for this well.

16-3. Consider the one-dimensional problem of a particle of mass m bound in a rectangular potential well:

$$V = V_o \qquad |x| > a$$

$$V = 0 \qquad |x| < a$$

Show that the two equations below are obtained by requiring that the wave functions are solutions of the Schrödinger equation which fulfill the required boundary conditions:

$$\alpha \cot \alpha a = \beta, \quad \text{or}$$

$$\alpha \tan \alpha a = + \beta, \quad \text{where}$$

$$\alpha = + \left(\frac{2mE}{\hbar^2} \right)^{1/2}$$

$$\beta = + \left[\frac{2m(V_o - E)}{\hbar^2} \right]^{1/2}$$

If $V_o a^2 = 4\hbar^2/2m$, estimate the energies of the ground state and first excited state. Sketch the wave functions of these states. How many bound states are there if $V_o a^2 < \frac{\pi^2 \hbar^2}{8m}$?

16-4. In Chapter 16 the momentum spread associated with a gaussian wave function was found. In general, the spatial width will not remain constant in time, however, but will spread out:

$$\phi(x) = K\, e^{-[a(t)\, x^2 + c(t)]}$$

Using the Schrödinger equation, show that for a free particle,

$$\frac{1}{a(t)} = \frac{1}{a_o} + \frac{2i\hbar}{m} t$$

What is c(t)? If the wave function describes an electron initially confined to a region 1 Å wide, how far will it be spread in 1 second?

Transform the wave function to momentum space; i.e., find the probability of finding the particle with a definite momentum p. How does the width of the momentum probability function vary with time? Show that the momentum spread found here is in agreement with the "velocity spread" found directly from the time dependence of the spatial wave function.

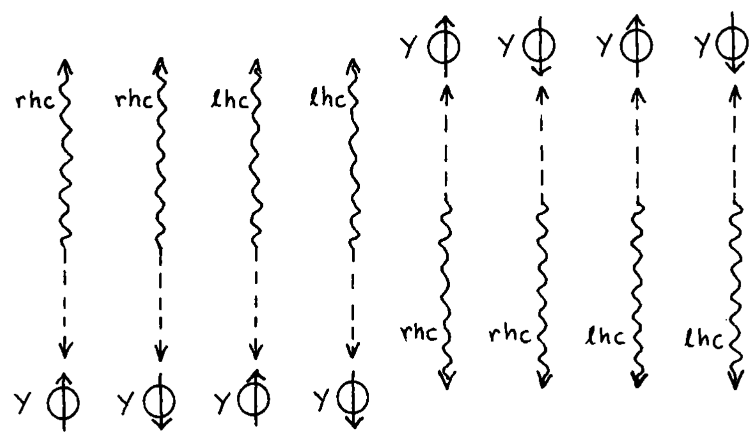
CHAPTER 18

18-1. A certain excited state of an atom has spin one and can lose its energy by emitting a photon and going to a state of spin zero. Consider an excited atom whose component of angular momentum along some z-axis is zero, and let $A(\theta)$ be the amplitude that it will emit a right-hand-circularly polarized photon into a small solid angle $\Delta\Omega$ in a direction at the angle θ with respect to the z-axis. How does $A(\theta)$ depend on θ?

18-2. The X, a spin 1/2 even parity particle, decays via the following scheme

$$X \rightarrow Y + \gamma$$

where the Y is a spin 1/2 even parity particle. If the X were polarized with its spin up along the +z axis, it could decay with the end products moving along the z axis in the following eight ways:



amplitude a b c d e f g h

The indicates a photon moving up, the arrow on the Y indicates the direction of its spin.

a) The amplitude for each of these final states is indicated below the drawing. Which amplitudes are necessarily zero?

b) Calculate the angular distribution of Y's polarized with their spins pointing in the same direction as their direction of motion when a group of X's polarized up along the z axis decays.

c) Calculate the angular distribution of all Y's, regardless of their polarizations, when a group of X's polarized up along +z decays.

d) Careful experiments have failed to show any deviation from a uniform angular distribution in this decay. What might be the physical reason for this?

18-3. The reaction

$$\gamma + p \rightarrow p^* \rightarrow p + \pi^o$$

is commonly studied at the CIT synchrotron. In this reaction p^* is an excited state of the proton which decays into a proton and π^o Over a certain range of photon energies the p^* is found to have a total angular momentum $j = 3/2$.

Assume that a beam of right hand circularly polarized photons at the right energy to give a p^* with $j = 3/2$ is incident in the +z direction on unpolarized protons. The angular distribution for this reaction can be analyzed as follows:

The photon plus proton have the amplitude $\underline{a}$ to form the p^* in the state $|j = 3/2, m = +1/2\rangle$ and the amplitude $\underline{b}$ to form the p^* in the state $|3/2, +3/2\rangle$.

The excited state of the proton, p^*, now decays into a neutral pion with zero spin and a proton moving in opposite directions. Let $\underline{f}$ be the amplitude for the proton to go along the +z axis with its spin up and let $\underline{g}$ be the amplitude for it to go along the +z axis with its spin down.

Explain why only $m = + 3/2$ and $m = + 1/2$ are allowed for the p^* and why only $m' = + 1/2$ and $m' = \;\; 1/2$ are allowed for the final state. (The m' refers to the direction of emission.) Predict the angular distribution of π^0's in terms of <u>a</u>, <u>b</u>, and θ. Assume $f = g$.

18-4. Consider elastic scattering of π^+ mesons by an unpolarized proton target. (The meson has spin zero; parity is conserved.) It is hypothesized that the scattering is dominated by a process in which the proton is excited to a state with $j = 3/2$ by absorption of the meson. ($j = 3/2$ is achieved by a combination of proton spin and meson-proton orbital angular momentum.) The meson is then re-emitted with the proton returning to its ground state.

Show that such a hypothesis predicts an angular distribution of scattered mesons proportional to $(1 + 3 \cos^2\theta)$.

18-5. The ground state of an atom has spin zero and even parity. The first excited state has spin 1 and unknown parity. Assume a supply of atoms in the first excited state, all with $m = +1$ in the z-direction, and consider photons emitted in transitions to the ground state.

a) If photons are detected regardless of their polarization, investigate whether their angular distribution can be used to determine the parity of the first excited state.

b) Show that measurement of the angular distributions of x' and y'-polarized photons can determine the unknown parity. (The z' axis is taken along the direction of photon emission, and is in the x-z plane.)